

# Developing Mathematical Habits of Mind with Origami

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## Abstract

Much has been written about the connections between origami, science, and mathematics. Connections appear in wide variety of contexts ranging from education programs (e.g., Geometria, a system developed by Miri Golan in Israel to teach young children about geometry through a sequence of folding and reflection activities) to engineering innovations that are based on folding strategies (e.g., folding space telescopes and unfolding solar arrays to use in space) to medical inventions (e.g., stents).

This presentation will focus on the synergies between mathematical habits of mind (MHoM), mathematics, and origami. When learning mathematics, it is important not only to learn mathematical facts and ideas, but to learn how to think in a mathematical way. MHoM can allow students to become comfortable with ill-posed and fuzzy problems, to see the benefit of systematizing and abstraction, and to look for and develop new ways of describing situations, as well as to help them develop genuinely mathematical ways of thinking. There are many problems in realms not usually defined as mathematical in which mathematical thinking or MHoM can spur creativity. We often talk about ways that mathematics can help solve a number of practical life problems, but most people never give mathematics a thought when working in more creative realms. If mathematical thinking is truly habitual, i.e., part of our skill set, then it will contribute to our approach to any problem. Students will be more willing to develop MHoM if they believe that these habits will be useful outside of the mathematics classroom.

Two examples that show different ways that MHoM can be developed within the context of origami are described below. Other examples may be included in the presentation if time allows.

### Cubes in a Holding Box

In this origami exercise students are taught to make stacking cubes (created by David Mitchell and Paul Jackson) and an open top box (created by Giovanni Baggi) to hold three cubes. After learning how to construct the stacking cubes and the box, students are assigned an exploration focused on figuring out what size rectangle is needed to make a box into which the cubes will nicely fit. Factors such as the amount of “wiggle room” needed to get the cubes in and out of the box are also considered. Students are then challenged to write a general “formula” for the paper size needed for a box for cubes of any size. Finally, students

are asked to write another formula relating the size of the paper used to make the cubes to the size of paper needed to make the box.

Students are required to draw pictures to solve this problem, including marking and labelling relevant information on the crease pattern for both the cubes and the box. One of the objectives here is to show that a formula does not have to begin abstractly with variables in an algebraic relationship, but can emerge from a visual analysis. Another objective is to begin to develop an idea of a mathematical model, including thinking about relevant factors to include, such as “wiggle room.” Questions such as, “Is ‘wiggle room’ needed?” “Does ‘wiggle room’ need to be quantified?” or “Does the ‘wiggle room’ have a general relationship to the size of the cubes?”

As the set of exercises progresses, cubes made in different ways (i.e., folded on lines parallel to edges or folded on lines parallel to the diagonal) are introduced. A new formula will need to be written for each different cube in order to show how on the size of the cube depends on the size of the paper used to make it.

Students are also challenged to raise their own questions and/or to think about other design changes that could be implemented in an interesting way.

### Bar Envelope

Students are taught to fold a one-piece envelope (created by Frances LeVangia) from a rectangle. The challenge here is to represent the size of the envelope in terms of the size of the starting sheet of paper. As students explore this relationship, the question arises about whether any shape envelope can actually be made. To answer this question, students are encouraged to experiment and to use variables to represent the two dimensions of the sheet of paper and of the resulting envelope. There are also modelling questions, similar to those described above for the stacking cubes and box, that should be explored. E.g, how much extra room is needed for an item to be placed nicely inside the envelope. This project has good potential for design innovations as well.

### Mathematical Habits of Mind Resulting From Origami Projects

- Curiosity about general properties and new uses of specific origami models
- Carefully observing characteristics of objects
- Forming symbolic representations, such as equations or formulas, of relationships within objects (e.g., dimensions of the envelope) or between origami models (e.g., cubes and box)
- Using crease patterns, drawings, observations, and materials to explore and generalize relationships